16A Linear transformations have rules of the form

$$(x, y) \to (ax + by, cx + dy)$$

or as vectors $\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$ why?

$$\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} ax \\ cx \end{bmatrix} + \begin{bmatrix} by \\ dy \end{bmatrix}$$

can split

$$\begin{bmatrix} x \\ y \end{bmatrix} \to x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix}$$

but remember...

$$x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix} \to x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix}$$

$$x\,\underset{\sim}{i} + y\,\underset{\sim}{j} \to x\,\underset{\sim}{m} + y\,\underset{\sim}{n}$$

this is a single vector say $\underset{\sim}{p}$ $\to$ this is a single vector say $\underset{\sim}{q}$

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a LT moves one vector (point) to another by moving its components (coordinates)

we have moved $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $\begin{bmatrix} a \\ c \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} b \\ d \end{bmatrix}$.

The multiples x, y stay the same.

(we are changing basis vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ into new ones $\underset{\sim}{m}$ and $\underset{\sim}{n}$)

which will transform any shape defined by them

Linear transformations do this for us

how do we transform in practice?

$$x\begin{bmatrix}1\\0\end{bmatrix} + y\begin{bmatrix}0\\1\end{bmatrix} \rightarrow x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix}$$

$$\begin{bmatrix}x\\0\end{bmatrix} + \begin{bmatrix}0\\y\end{bmatrix} \rightarrow \begin{bmatrix}ax\\cx\end{bmatrix} + \begin{bmatrix}by\\dy\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix} \rightarrow \begin{bmatrix}ax + by\\cx + dy\end{bmatrix}$$

we will define matrix multiplication as the op that does this for us

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}ax + by\\cx + dy\end{bmatrix}$$

$$= x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix}$$

from the top line

this is another way of thinking about matrix multiplication

Remember this from three slides ago?

$$x\begin{bmatrix}1\\0\end{bmatrix} + y\begin{bmatrix}0\\1\end{bmatrix} \rightarrow x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix}$$

$$x\underset{\sim}{i} + y\underset{\sim}{j} \rightarrow x(a\underset{\sim}{i} + c\underset{\sim}{j}) + y(b\underset{\sim}{i} + d\underset{\sim}{j})$$

I can move $x\underset{\sim}{i} + y\underset{\sim}{j}$ by moving its components

$$x\underset{\sim}{i} \longrightarrow x(a\underset{\sim}{i} + c\underset{\sim}{j})$$

since the scalar is the same, think of it as moving

$$\underset{\sim}{i} \longrightarrow a\underset{\sim}{i} + c\underset{\sim}{j}$$

$$\begin{bmatrix}1\\0\end{bmatrix} \longrightarrow \begin{bmatrix}a\\c\end{bmatrix}$$

similarly

$$y\underset{\sim}{j} \rightarrow y(b\underset{\sim}{i} + d\underset{\sim}{j})$$

$$\underset{\sim}{j} \rightarrow b\underset{\sim}{i} + d\underset{\sim}{j}$$

$$\begin{bmatrix}0\\1\end{bmatrix} \rightarrow \begin{bmatrix}b\\d\end{bmatrix}$$

Remember this from three slides ago?

$$x\begin{bmatrix}1\\0\end{bmatrix} + y\begin{bmatrix}0\\1\end{bmatrix} \rightarrow x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix}$$

$$x\underset{\sim}{i} + y\underset{\sim}{j} \longrightarrow x(a\underset{\sim}{i} + c\underset{\sim}{j}) + y(b\underset{\sim}{i} + d\underset{\sim}{j})$$

I can move $x\underset{\sim}{i} + y\underset{\sim}{j}$ by moving its components

$$x\underset{\sim}{i} \longrightarrow x(a\underset{\sim}{i} + c\underset{\sim}{j})$$

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similarly

$$y\underset{\sim}{j} \rightarrow b\underset{\sim}{i} + d\underset{\sim}{j}$$

$$\underset{\sim}{j} \rightarrow b\underset{\sim}{i} + d\underset{\sim}{j}$$

$$\begin{bmatrix}0\\1\end{bmatrix} \rightarrow \begin{bmatrix}b\\d\end{bmatrix}$$

So the transformation

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}ax+by\\cx+dy\end{bmatrix}$$

has the effect of moving $\begin{bmatrix}1\\0\end{bmatrix}$ to $\begin{bmatrix}a\\c\end{bmatrix}$

and $\begin{bmatrix}0\\1\end{bmatrix}$ to $\begin{bmatrix}b\\d\end{bmatrix}$

check: $\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}a\\c\end{bmatrix}$

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}b\\d\end{bmatrix}$$

relate this to the unit square

Remember this from three slides b4?

$$x\begin{bmatrix}1\\0\end{bmatrix} + y\begin{bmatrix}0\\1\end{bmatrix} \rightarrow x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix}$$

$$x\underset{\sim}{i} + y\underset{\sim}{j} \rightarrow x(a\underset{\sim}{i} + c\underset{\sim}{j}) + y(b\underset{\sim}{i} + d\underset{\sim}{j})$$

$$x\underset{\sim}{i} + y\underset{\sim}{j} \rightarrow x\underset{\sim}{m} + y\underset{\sim}{n} \quad \text{where } \underset{\sim}{m} = a\underset{\sim}{i} + c\underset{\sim}{j}$$

$$\underset{\sim}{n} = b\underset{\sim}{i} + d\underset{\sim}{j}$$

$\begin{bmatrix} \leftarrow \\ \leftarrow \end{bmatrix}$ horizontal direction $\underset{\sim}{i}$

vertical direction $\underset{\sim}{j}$

"representation"

even though we have changed basis vectors from $\underset{\sim}{i}$ and $\underset{\sim}{j}$ to $\underset{\sim}{m}$ and $\underset{\sim}{n}$, the scalar multiples x and y remain the same

this is a fundamental property of linear transformations

Remember this from three slides b4?

$$x\begin{bmatrix}1\\0\end{bmatrix} + y\begin{bmatrix}0\\1\end{bmatrix} \rightarrow x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix}$$

$$x\underset{\sim}{i} + y\underset{\sim}{j} \rightarrow x(a\underset{\sim}{i} + c\underset{\sim}{j}) + y(b\underset{\sim}{i} + d\underset{\sim}{j})$$

$$x\underset{\sim}{i} + y\underset{\sim}{j} \rightarrow x\underset{\sim}{m} + y\underset{\sim}{n}$$

where $\underset{\sim}{m} = a\underset{\sim}{i} + c\underset{\sim}{j}$

$\underset{\sim}{n} = b\underset{\sim}{i} + d\underset{\sim}{j}$

So $\begin{bmatrix}a & b\\c & d\end{bmatrix}$ has transformed red to pink