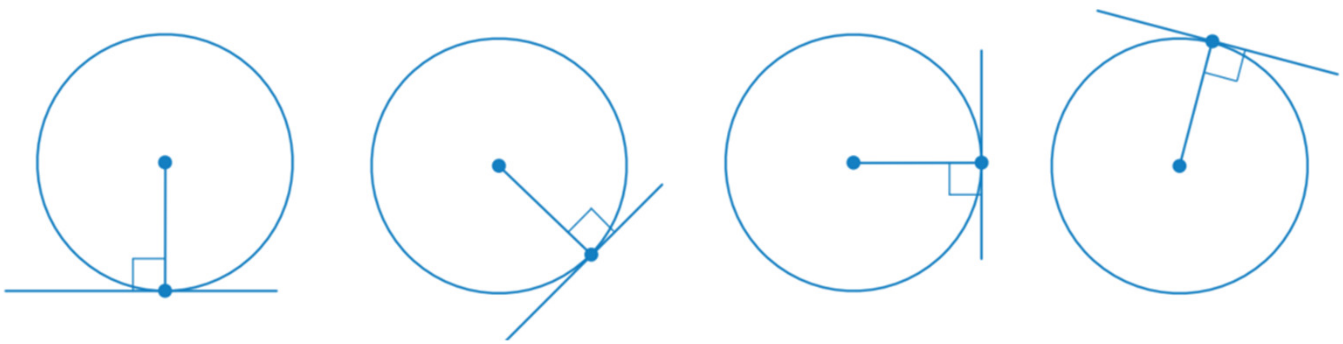
YEAR 11 SPECIALIST: CIRCLE GEOMETRY

8B Tangents & Secants

Theorem 5: Tangent is perpendicular to radius

A tangent to a circle is perpendicular to the radius drawn from the point of contact.

The angle between a tangent and the radius drawn at the point of contact is a right angle.



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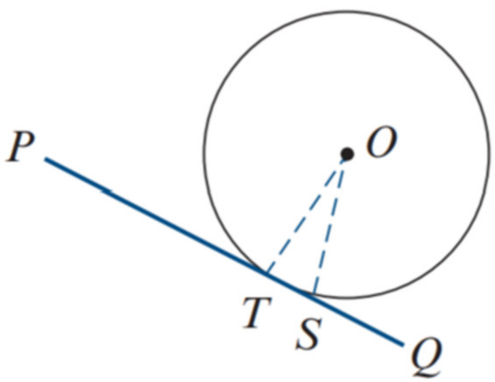


proof by contradiction

Let T be the point of contact of tangent PQ

Suppose $\angle OTP$ is NOT a right angle

Let S be a point on PQ, not S, such that OSP is a right angle

$\therefore$ $\triangle OST$ has a right angle at S

thus $OT > OS$ as OT is the hypotenuse of $\triangle OST$

This implies S is inside the circle as OT is a radius

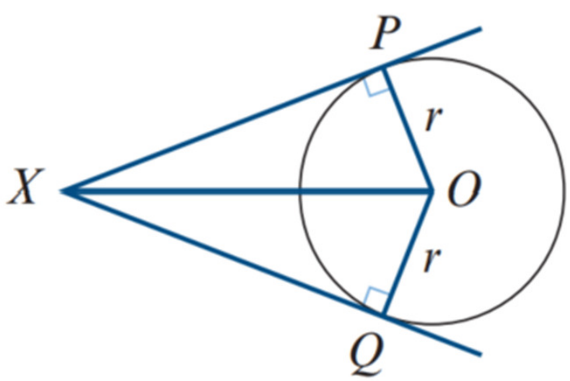
Thus a line through T&S must e a secant

But PQ is a tangent; this is a contradiction

$\therefore$ $\angle OTP$ is a right angle

Theorem 6: Two tangents from the same point

The two tangents drawn from an external point to a circle are the same length.



Theorem 6: Two tangents from the same point

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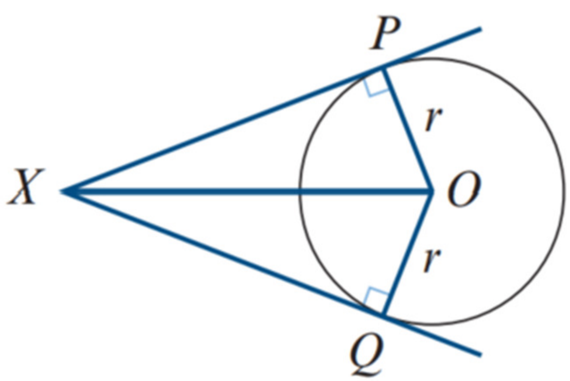


$\angle XPO = \angle XQO = 90^\circ$ (angle formed by a tangent and a radius is 90°)

$XO = XO$ (shared hypotenuse of $\triangle XPO$ & $\triangle XQO$)

$OP = OQ$ (radii of the circle)

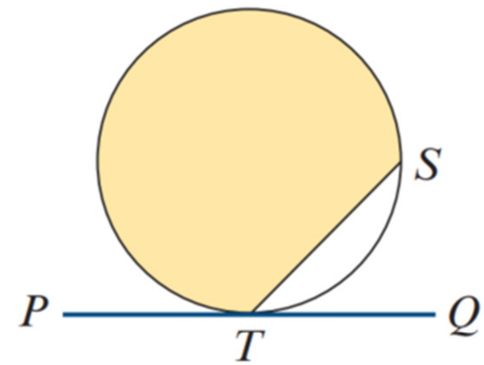
$\therefore \triangle XPO \equiv \triangle XQO$ (RHS congruency)

Thus $XP = XQ$

The alternate segment theorem

In the diagram:

- The shaded segment is called the **alternate segment** in relation to $\angle STQ$.
- The unshaded segment is alternate to $\angle STP$.



Theorem 7: Alternate segment theorem

The angle between a tangent and a chord drawn from the point of contact is equal to any angle in the alternate segment.

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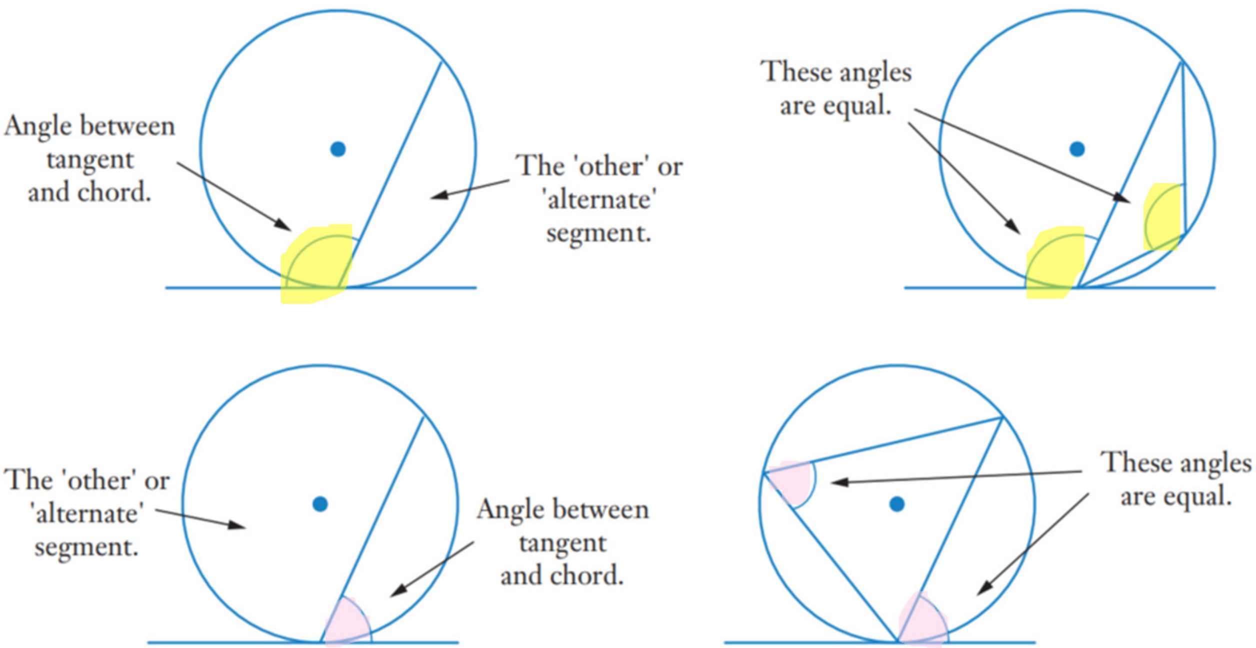

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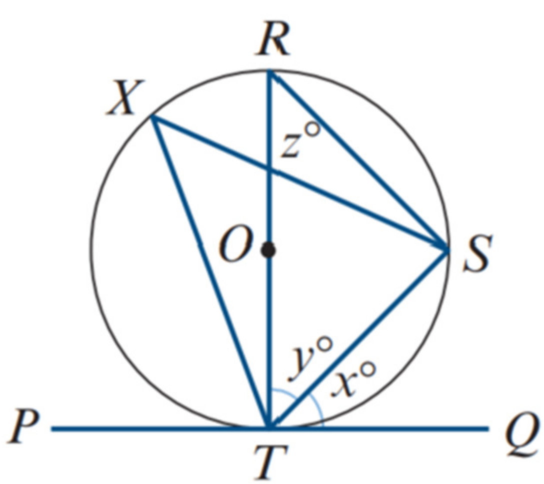


let $\angle STQ = x°$, $\angle RTS = y°$, $\angle TRS = z°$

RT is a diameter

$\angle RST = 90°$ (angle subtended by a diameter)

$\therefore y + z = 90$

also, $\angle RTQ = 90°$ (tangent $\perp$ radius)

$\therefore x + y = 90°$

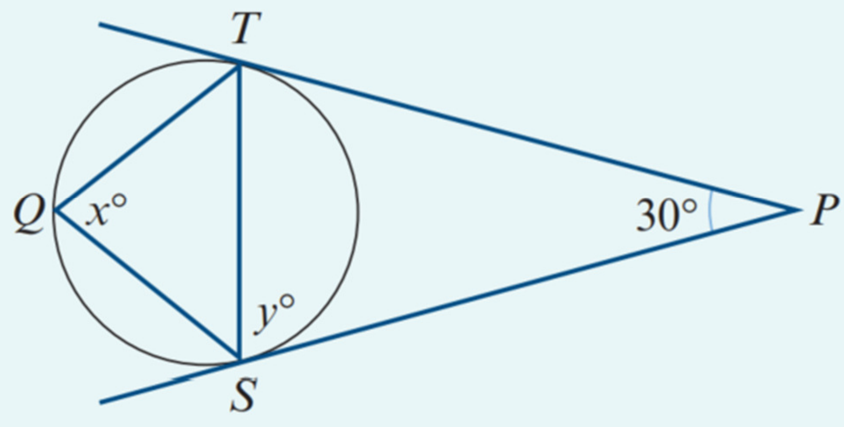
Thus $x = z$

But $\angle TXS$ is in the same segment as $\angle TRS$

$\therefore \angle TXS = x°$

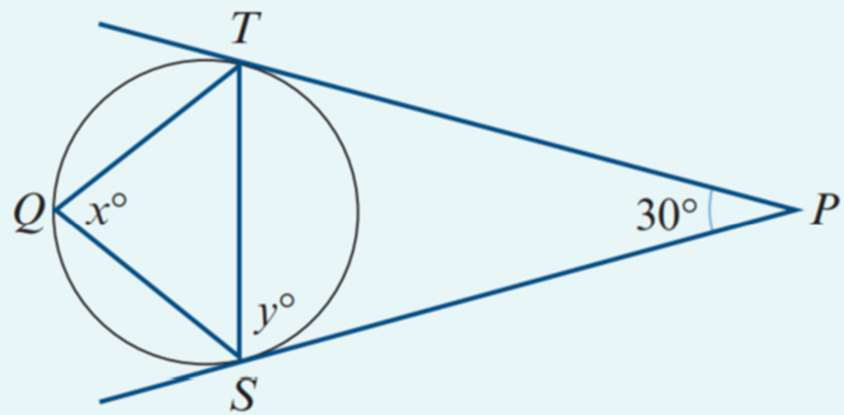
Example 4

Find the magnitudes of the angles x and y in the diagram.



Example 4

Find the magnitudes of the angles x and y in the diagram.



Solution

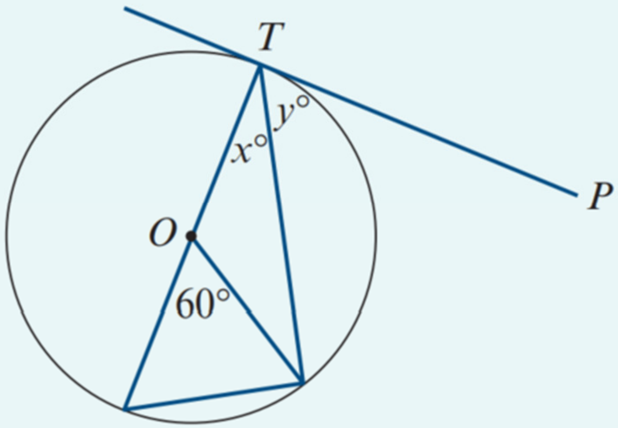
Triangle PST is isosceles (Theorem 6, two tangents from the same point).

Therefore $\angle PST = \angle PTS$ and so $y = 75$.

The alternate segment theorem gives $x = y = 75$.

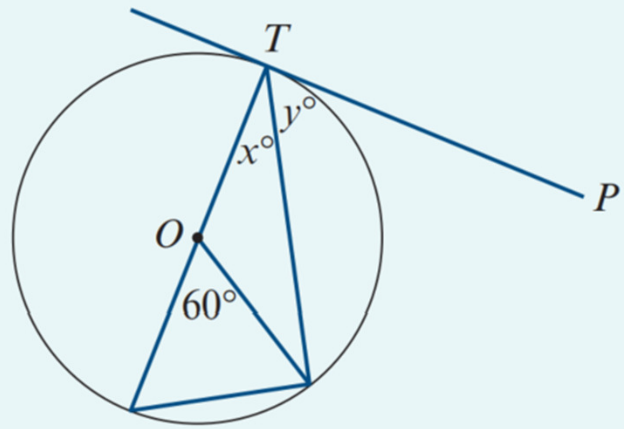
Example 5

Find the values of x and y, where PT is tangent to the circle centre O.



Example 5

Find the values of x and y, where PT is tangent to the circle centre O.

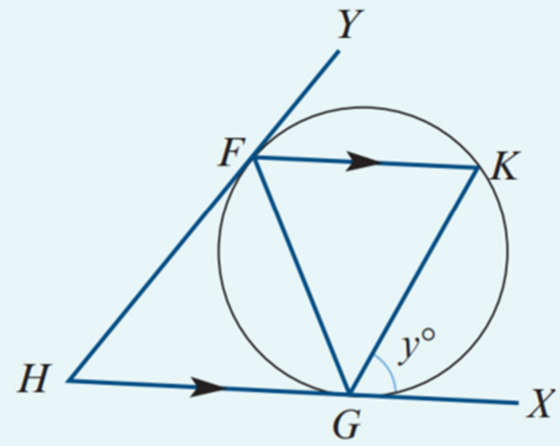


Solution

$x = 30$ as the angle at the circumference is half the angle subtended at the centre, and so $y = 60$ as $\angle OTP$ is a right angle.

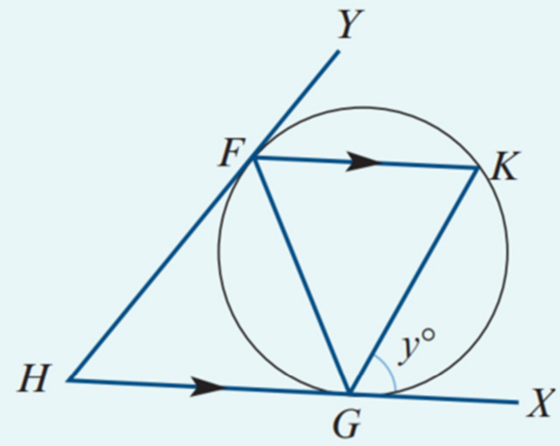
Example 6

The tangents to a circle at F and G meet at H. A chord FK is drawn parallel to HG. Prove that triangle FGK is isosceles.



Example 6

The tangents to a circle at F and G meet at H. A chord FK is drawn parallel to HG. Prove that triangle FGK is isosceles.



Solution

Let $\angle XGK = y^\circ$.

Then $\angle GFK = y^\circ$ (alternate segment theorem) and $\angle GKF = y^\circ$ (alternate angles).

Therefore triangle FGK is isosceles with $FG = KG$.