Rossmoyne Senior High School

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures | | | | | | | | |

In words ______________________

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Your name ______________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	91	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (91 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(6 marks)**

Four figure numbers are to be formed from the digits $1, 2, 3, 4, 5, 6$.

(a) How many different four figure numbers can be formed

(i) if repetition of digits is allowed? (1 mark)

Solution
$6^4 = 1296$
Specific behaviours
✓ correct number

(ii) without repetition of digits? (1 mark)

Solution
$6 \times 5 \times 4 \times 3 = 360$
Specific behaviours
✓ correct number

(b) How many of the numbers without repetition are greater than 5423? (3 marks)

Solution
Start with 542: $1 \times 1 = 1$ Start with 546 or 543: $2 \times 3 = 6$ Start with 56: $1 \times 4 \times 3 = 12$ Start with 6: $1 \times 5 \times 4 \times 3 = 60$ Total numbers: $1 + 6 + 12 + 60 = 79$
Specific behaviours
✓ splits into cases ✓ correct count for at least two cases ✓ correct number

(c) How many of the numbers without repetition are less than 5423? (1 mark)

Solution
$360 - 79 - 1 = 280$
Specific behaviours
✓ correct number

Question 9 **(6 marks)**

A classroom has a barrel containing a large number of red crayons and green crayons.

(a) 12 crayons are randomly drawn from the barrel and placed, in the order, on a table. In terms of their colours, how many different orders are possible? (1 mark)

Solution
$2^{12} = 4096$
Specific behaviours
✓ correct number

(b) An empty box is to be filled with some of the crayons. What is the smallest number of crayons that should be placed in the box to guarantee that it contains at least 6 red or at least 5 green crayons? Justify your answer. (2 marks)

Solution
Take two pigeonholes and fill one with 5 red and the other with 4 green. When one more crayon is chosen and placed in its pigeonhole, then one of the conditions will be met and so $5 + 4 + 1 = 10$ is the smallest number of crayons that should be placed in the box.
Specific behaviours
✓ correct smallest number ✓ justification

(c) Each of the 11 students in the classroom will choose 4 crayons from the barrel. Use the pigeonhole principle to prove that at least 3 of the students will choose the same colour combination of crayons. (3 marks)

Solution
All possible combinations of crayons represent the pigeonholes, so that there are 5 pigeonholes ($0, 1, 2, 3$ or 4 reds in selection). The pigeons are the selections made by each student, so that there are 11 pigeons. Using the pigeonhole principle, there will be at least $\lceil 11 \div 5 \rceil = 3$ students who choose the same colour combination.
Specific behaviours
✓ relates pigeonholes and pigeons to context ✓ correctly identifies number of pigeonholes ✓ completes proof using pigeonhole principle

Question 10 **(8 marks)**

Relative to boat O at anchor in a lake, four buoys A, B, C and D have the following position vectors (with distances in metres):

$$\overrightarrow{OA} = (210, -935), \quad \overrightarrow{OB} = (90, -200), \quad \overrightarrow{OC} = (330, 360), \quad \overrightarrow{OD} = (390, -515).$$

(a) Prove that the quadrilateral with vertices $ABCD$ is a trapezium, but not a parallelogram. (5 marks)

Solution
Displacement vectors for all four sides are $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-120, 735)$ $\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = (-60, 875)$ $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (240, 560)$ $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = (180, 420)$ Using $\mathbf{i}$-coefficients, $\frac{240}{180}\overrightarrow{AD} = (240, 560) = \overrightarrow{BC}$ and hence $\overrightarrow{AD}$ is parallel to $\overrightarrow{BC}$. Also, $\frac{120}{60}\overrightarrow{DC} = (-120, 1750) \neq \overrightarrow{AB}$ and hence $\overrightarrow{DC}$ is not parallel to $\overrightarrow{AB}$. Hence $ABCD$ has just one pair of parallel sides and thus is a trapezium but not a parallelogram.
Specific behaviours
✓ calculates correct displacement vectors for at least one side ✓ calculates correct displacement vectors for all sides ✓ clearly shows $\overrightarrow{AD}$ is parallel to $\overrightarrow{BC}$. ✓ clearly shows $\overrightarrow{DC}$ is not parallel to $\overrightarrow{AB}$ ✓ uses results to justify $ABCD$ is a trapezium but not a parallelogram

(b) Boat X motors directly from D to B with a constant velocity taking 2 minutes and 30 seconds. Determine the velocity in component form, and hence the speed, of boat X. (3 marks)

Solution
Displacement vector $\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD} = (-300, 315)$ m. Hence velocity vector $\mathbf{v}_{DB} = \overrightarrow{DB} \div 150 = (-2, 2.1)$ m/s. Hence speed $= \lvert\mathbf{v}_{DB}\rvert = 2.9$ m/s.
Specific behaviours
✓ displacement vector ✓ velocity vector ✓ speed, with units.

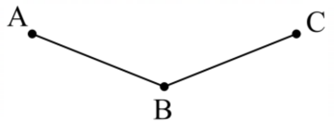
Question 11 **(8 marks)**

(a) Which of the following statements are false?
Justify your answer by giving a suitable counterexample(s). (3 marks)

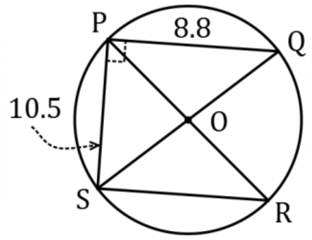
A $\forall x, y \in \mathbb{Z}\ \ x^2 < y^2 \Rightarrow x < y$

B If $\overline{AB} = \overline{BC} \Leftrightarrow B$ is the midpoint of AC

C $\forall x\, \exists y: y > x$

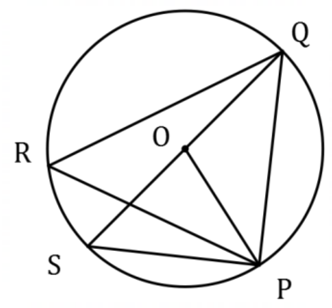
Solution
A is false. If $x = 1$ and $y = -2$, then $x^2 < y^2$ is true, but $x < y$ is false.
B is false. For example: 
C is true. There is always a number which is larger than another.
Specific behaviours
✓ Determines at least one statement which is false.
✓ Determines both false statements.
✓ Provides a counterexample for both false statements.

(b) Points P, Q, R and S lie in order on the circumference of the circle with centre O so that $PQ = 8.8$ cm, $PS = 10.5$ cm, and PR and QS are diameters. Determine, with brief reasons and to the nearest degree, the sizes of $\angle PQS$, $\angle PRS$, $\angle POS$ and $\angle RPS$. (5 marks)

Solution	
$\angle PQS = \tan^{-1}\left(\frac{10.5}{8.8}\right) = 50°$ $\angle PRS = \angle PQS = 50°$ (angle on same arc) $\angle POS = 2\angle PQS = 100°$ (angle at centre) $\angle RPS = 90° - 50° = 40°$ (angle in semicircle) or $(180° - 100°) \div 2 = 40°$ (Isos. $\triangle$)	
Specific behaviours	
✓ labelled diagram showing diameters, chords, lengths ✓✓✓✓ calculates each angle, with reasoning NB: Give max 2 out of 4 marks for the "calculations of angles" if students were not able to calculate the 50° but provided correct reasoning. (1 mark for at least 1 correct relationship, 2 marks for 3 correct relationships)	

Question 12 **(8 marks)**

(a) In the diagram (not to scale), points P, Q, R and S lie on a circle with centre O and diameter QS.



If $\angle QRP = 59°$, determine the size of

(i) $\angle QOP$. (1 mark)

(ii) $\angle QSP$. (1 mark)

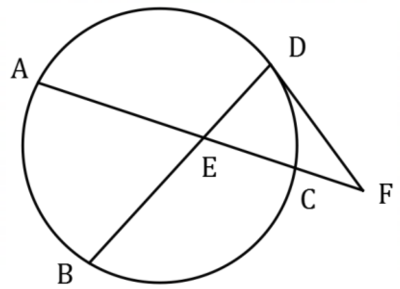
(iii) $\angle SQP$. (1 mark)

(iv) $\angle POS$. (1 mark)

Solution
$\angle QOP = 2\angle QRP = 118°$
$\angle QSP = \angle QRP = 59°$
$\angle SQP = 90° - \angle QSP = 31°$
$\angle POS = 2\angle SQP = 62°$
Specific behaviours
✓✓✓✓ each correct angle

(b) In the diagram (not to scale), FD is a tangent to the circle at D. Secant AF cuts chord BD at E, and the circle at C.

$FC = 4$ cm, $CA = 21$ cm, $DF = EF$, and BE is 2 cm longer than DF.



Determine the length of DE. (4 marks)

Solution
Secant-tangent property:
$CF \times AF = DF^2$
$(21 + 4) \times 4 = DF^2$
$DF = 10$ cm
Intersecting chords property:
$BE \times DE = AE \times CE$
$(10 + 2) \times DE = \big(21 - (10 - 4)\big)(10 - 4)$
$12DE = 15 \times 6$
$DE = 7.5$ cm
Specific behaviours
✓ correct use of secant-tangent property
✓ length DF
✓ correct use of intersecting chords property
✓ length DE

Question 13 **(8 marks)**

Two vectors are $\mathbf{a} = \begin{pmatrix} x \\ -5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 6 \\ y \end{pmatrix}$, where x and y are constants.

(a) When $x = 11$ and $y = 7$, determine

(i) $\mathbf{a} \cdot \mathbf{b}$. (1 mark)

Solution
$\begin{pmatrix} 11 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 7 \end{pmatrix} = 31$
Specific behaviours
✓ correct value

(ii) a unit vector in the same direction as $\mathbf{a} - \mathbf{b}$, in exact form. (2 marks)

Solution
$\mathbf{u} = \mathbf{a} - \mathbf{b} = \begin{pmatrix} 11 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ 7 \end{pmatrix} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}, \qquad \hat{\mathbf{u}} = \begin{pmatrix} 5/13 \\ -12/13 \end{pmatrix}$
Specific behaviours
✓ indicates $\mathbf{a} - \mathbf{b}$ ✓ correct unit vector

(iii) the angle between the directions of the unit vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, rounded to one decimal place. (2 marks)

Solution
NB the direction of a vector and its unit vector are the same. Using CAS, $\theta = 73.843° \approx 73.8°$ to one dp.
Specific behaviours
✓ indicates method ✓ correct angle, as required

(b) Determine the value of x and the value of y when $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, $x < y$, and $|\mathbf{a} + \mathbf{b}| = 12.2$. (3 marks)

Solution
$\mathbf{a} \cdot \mathbf{b} = 0 \Rightarrow 6x - 5y = 0$ $\mathbf{a} + \mathbf{b} = \begin{pmatrix} x + 6 \\ y - 5 \end{pmatrix}$ $\lvert\mathbf{a} + \mathbf{b}\rvert = 12.2 \Rightarrow (x + 6)^2 + (y - 5)^2 = 12.2^2$ Solving simultaneously for solution where $x < y$: $x = 6, \qquad y = 7.2$
Specific behaviours
✓ obtains equation using scalar product ✓ obtains equation using sum of magnitudes ✓ states correct solution set

Question 14 **(7 marks)**

(a) Consider the letters in the word EXTRAVAGANCE. Determine the number of different

(i) combinations of 4 letters chosen from the consonants in the word. (1 mark)

Solution
$\binom{7}{4} = 35$
Specific behaviours
✓ correct number

(ii) permutations of all the letters in the word. (2 marks)

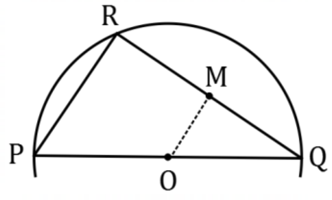
Solution
$n = \frac{12!}{3!\,2!} = 39\,916\,800$
Specific behaviours
✓ reasonable attempt to deal with repeated letters ✓ correct number (must be numerical not factorial)

(b) Four-digit pin codes such as 0852 are made by randomly choosing four different digits from those in the number $234\,567\,890$. Determine the fraction of all such possible pin codes that start with 2 or end in 90. (4 marks)

Solution
If T are codes that start with 2, and N are codes that end in 90, then $n(T) = 1 \times 8 \times 7 \times 6 = 336$ $n(N) = 7 \times 6 \times 1 \times 1 = 42$ $n(N \cap T) = 1 \times 6 \times 1 \times 1 = 6$ $n(N \cup T) = 336 + 42 - 6 = 372$ Total number of codes is ${}^{9}P_{4} = 3024$. Hence fraction of all codes is $\frac{372}{3024} = \frac{31}{252}$
Specific behaviours
✓ calculates $n(T), n(N)$ ✓ calculates $n(N \cap T)$ ✓ uses inclusion-exclusion for $n(N \cup T)$ ✓ number of all possible codes and correct fraction (fraction doesn't need to be simplified)

Question 15 **(8 marks)**

(a) The vertices of triangle PQR lie on a circle with centre O so that PQ is a diameter. The midpoint of QR is M. Prove that OM is parallel to PR. (4 marks)

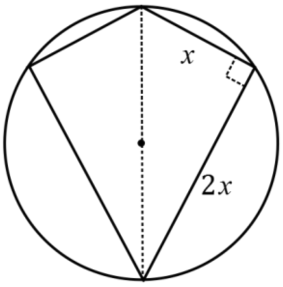
Solution	
Angle in semicircle: $\angle PRQ = 90°$. Line from O bisecting chord is perpendicular to chord, so $\angle OMQ = 90°$. Hence $\angle OMQ = \angle PRQ$ and since they are corresponding angles then OM is parallel to PR.	

Specific behaviours

✓ diagram
✓ reasons for $\angle PRQ = 90°$
✓ reasons for $\angle OMQ = 90°$
✓ completes proof

NB A vector proof is also acceptable, where $\overrightarrow{PR} = 2\overrightarrow{OM}$ is shown
See page 15

(b) The vertices of a kite lie on the circumference of a circle. Each longer side of the kite is twice the length of the adjacent shorter side. If the area of the kite is 18 cm², determine the radius of the circle. (4 marks)

Solution	
RHS of kite is right triangle (angle in semicircle). Hence: $A = \frac{1}{2}(2x)(x) \times 2$ $2x^2 = 18$ $x = 3$ Hence $(2r)^2 = 6^2 + 3^2$ $r = \frac{3\sqrt{5}}{2} \approx 3.35 \text{ cm}$	

Specific behaviours

✓ reasonable diagram
✓ uses angles in semicircle property to form area equation
✓ solves equation
✓ correct radius (accept decimal answer if it is to at least 2 dp)

Question 16 **(6 marks)**

Consider the identity ${}^{x}C_{y} \times {}^{y}C_{z} = {}^{x}C_{z} \times {}^{(x-z)}C_{(y-z)}$.

(a) With $x = 4, y = 2$ and $z = 1$, show that the left and right sides of the identity are equal. (1 mark)

Solution
$LHS = \binom{4}{2}\binom{2}{1} = 6 \times 2, \qquad RHS = \binom{4}{1}\binom{3}{1} = 4 \times 3 = 12$ $\therefore LHS = 12 = RHS$
Specific behaviours
✓ correctly shows substitution and simplification (Must show that LHS=RHS=12. Comment only about need to show individual terms)

(b) State all necessary restrictions on x, y and z for the identity to exist and to be valid. (2 marks)

Solution
$x, y, z \in \mathbb{Z}, \qquad x \geq y \geq z \geq 0$
Specific behaviours
✓ states at least one restriction (e.g., $y \geq 0$) ✓ states all restrictions (Give 1 of 2 marks if all restrictions, but uses $>$ instead of $\geq$)

Restrictions
• Values must be positive • Values must be integers. • $x \geq y \geq z$

(c) Prove the identity is always true, subject to all necessary restrictions. (3 marks)

Solution

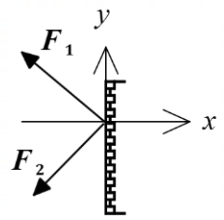
$$\begin{aligned} RHS &= {}^{x}C_{z} \times {}^{x-z}C_{y-z} \\ &= \frac{x!}{z!\,(x-z)!} \times \frac{(x-z)!}{(y-z)!\,(x-z-y+z)!} \\ &= \frac{x!\,(x-z)!}{z!\,(x-z)!\,(y-z)!\,(x-y)!} \\ &= \frac{x!}{z!\,(y-z)!\,(x-y)!} \\ &= \frac{x!\,y!}{z!\,(y-z)!\,y!\,(x-y)!} \\ &= \frac{x!}{y!\,(x-y)!} \times \frac{y!}{z!\,(y-z)!} \\ &= {}^{x}C_{y} \times {}^{y}C_{z} \\ &= LHS \end{aligned}$$

Specific behaviours

✓ correctly obtains factorial expression for RHS
✓ cancels out $(x-z)!$ terms and introduces $y!$ terms
✓ completes proof

Question 17 **(9 marks)**

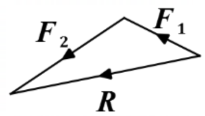
The diagram at right, not to scale, shows forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting in the same vertical plane on a small hook fixed to a vertical wall. $\mathbf{F}_1$ has magnitude 147 N and acts at an angle of elevation of $22°$ and $\mathbf{F}_2$ has magnitude 195 N and acts at an angle of depression of $42°$.



The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.

(a) Sketch a triangle to show the relationship between $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{R}$. (1 mark)

Solution



Specific behaviours

✓ nose-to-tails force vectors and completes triangle with resultant

(b) Determine, with reasoning, the magnitude of $\mathbf{R}$ and the acute angle it makes with the wall. (5 marks)

Solution

Angle in triangle between forces is $180° - 22° - 42° = 116°$.

$$|\mathbf{R}| = 147^2 + 195^2 - 2(147)(195)\cos 116°$$
$$= 291.15 \approx 291 \text{ N}$$

Let θ be the angle between $\mathbf{F}_1$ and $\mathbf{R}$:

$$\frac{\sin\theta}{195} = \frac{\sin 116°}{291.15}$$
$$\theta = 37.0°$$

Hence acute angle with wall is $90° + 22° - 37° = 75°$.

OR $180° - 37° - (90° - 22°) = 75°$

Specific behaviours

✓ correct angle between forces (*shown here or in (a)*)
✓ expression using cosine rule with magnitude
✓ calculates magnitude
✓ expression using sine rule with angle
✓ calculates angle with horizontal

(c) The wall exerts a force on the hook of equal magnitude to $\mathbf{R}$ but in the opposite direction. Express this force using unit vectors $\mathbf{i}$ and $\mathbf{j}$. (3 marks)

Solution

Angle $\mathbf{R}$ makes with x-axis is $-90° - 75° = -165°$.

$$\mathbf{R} = 291.15(\cos(-165°), \sin(-165°))$$
$$= -281.2\mathbf{i} - 75.4\mathbf{j}$$

Hence force exerted by wall is $281.2\mathbf{i} + 75.4\mathbf{j}$.

Specific behaviours

✓ indicates angle of resultant with x-axis, or similar
✓ converts into component form
✓ correctly reverses direction

Alternate Solution

Angle force of the wall, $\mathbf{W}$, makes with x-axis is $90° - 75° = 15°$.

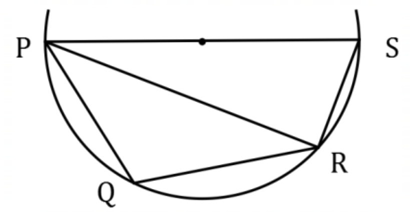
$$\mathbf{W} = 291.15(\cos(15°), \sin(15°))$$
$$= 281.2\mathbf{i} + 75.4\mathbf{j}$$

Specific behaviours

✓ indicates angle of $15°$
✓✓ correctly obtains vector in component form.

Question 18 **(8 marks)**

(a) Points P, Q, R and S lie on an arc of a circle as shown, so that PS is a diameter.



Let $\lambda = \angle SPR$ and $\mu = \angle PQR$.

Determine in simplest form the relationship between λ and μ. Justify your answer (3 marks)

Solution

Angles in semicircle:

$$\angle SPR + \angle PSR = 90° \rightarrow \angle PSR = 90° - \lambda$$

Opposite angles in cyclic quadrilateral:

$$\angle PQR + \angle PSR = 180° \rightarrow \angle PSR = 180° - \mu$$

Hence $90° - \lambda = 180° - \mu \rightarrow \mu - \lambda = 90°$.

Specific behaviours

✓ uses angles in semicircle property
✓ uses opposite angles property
✓ simplified relationship

(b) Tangents from E touch a circle at A and D. Diameter DC and tangent EA are both extended to meet at B. Prove that $\angle DEA = 2\angle CAB$. (5 marks)

Solution

1. Angles in semicircle:

$$\angle CAD = 90°$$

2. Straight angle:

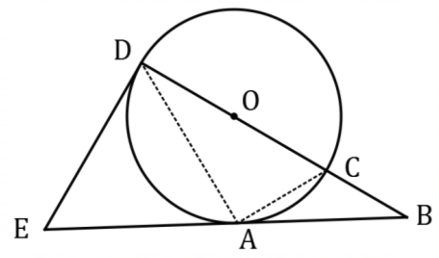
$$\angle EAD = 180° - 90° - \angle CAB$$
$$= 90° - \angle CAB$$

3. Tangents from point form isosceles triangle:

$$\angle DEA = 180° - 2\angle EAD$$
$$= 180° - 2(90° - \angle CAB)$$
$$= 2\angle CAB$$

Hence $\angle DEA = 2\angle CAB$.

See page 16 for Alternate Solution



1. Angles in alternate segments:

$$\angle ADC = \angle CAB$$

2. Tangent-radius is right angle:

$$\angle EDA = 90° - \angle ADC$$
$$= 90° - \angle CAB$$

Specific behaviours

✓ diagram
✓ shows (1) - that $\angle CAD = 90°$ or $\angle ADC = \angle CAB$
✓ shows (2) - that $\angle EAD$ or $\angle EDA = 90° - \angle CAB$
✓ shows (3) - that $\angle DEA = 2\angle CAB$
✓ clear reasoning throughout

Question 19 **(9 marks)**

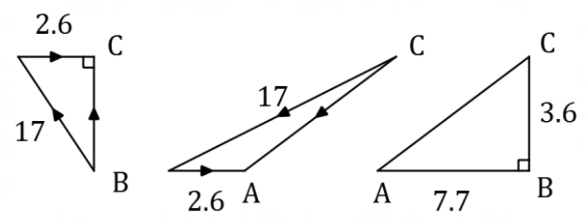
A small boat has a cruising speed of 17 km/h in still water. The boat leaves point A at $8{:}30$ am and travels to point B, 7.7 km due east of A, where it turns and travels to point C, 3.6 km due north of B. The boat then returns to A. A current of 2.6 km/h runs in an easterly direction throughout the area, during the entire period of travel.
Determine the time at which the boat is expected to return to A.

Solution

Time for leg AB: $t_{AB} = 7.7 \div (17 + 2.6) = 0.3929$ h.

Resultant Speed for leg BC: $s_{BC} = \sqrt{17^2 - 2.6^2} = 16.8$ km/h.

Time for leg BC: $t_{BC} = 3.6 \div 16.8 = 0.2143$ h.



$$\angle A = \tan^{-1}\left(\frac{3.6}{7.7}\right) = 25.0576° \rightarrow 180° - \angle A = 154.9424°$$

Resultant Speed for leg CA:

$$17^2 = s_{CA}^2 + 2.6^2 - 2(s_{CA})(2.6)\cos 154.9424°$$
$$s_{CA} = 14.61$$

Distance CA: $CA = \sqrt{3.6^2 + 7.7^2} = 8.5$ km
Time for leg CA: $t_{CA} = 8.5 \div 14.61 = 0.5818$

Total time: $t = 0.3929 + 0.2143 + 0.5818 = 1.189 = 1{:}11'20''$ h.
Boat will return to A at $8{:}30 + 1{:}11 = 9{:}41$ am.

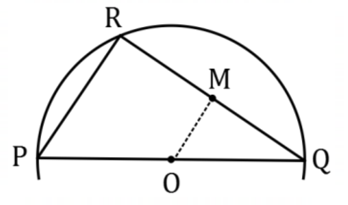
Specific behaviours

✓ time for leg AB
✓ speed for leg BC
✓ time for leg BC
✓ correct diagram for last leg (centre diagram)
✓ calculates angle in last leg triangle
✓ indicates correct use of trig to solve for s_{CA}
✓ speed for leg CA
✓ distance and time for leg CA
✓ correct return time

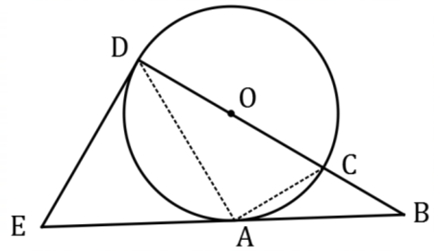
End of questions

Supplementary page

Question number: _________

Alternate Solution to Q15(a) [Vector Method]	
$\overrightarrow{OM} = \overrightarrow{QM} - \overrightarrow{QO}$ $= \frac{1}{2}\overrightarrow{QR} - \frac{1}{2}\overrightarrow{QP}$ $= \frac{1}{2}\left(\overrightarrow{QR} - \overrightarrow{QP}\right)$ $= \frac{1}{2}\overrightarrow{PR}$ Since $\overrightarrow{OM}$ is a scalar multiple of $\overrightarrow{PR}$, then OM is parallel to PR.	
Specific behaviours	
✓ diagram ✓ expression for $\overrightarrow{OM}$ in terms of $\overrightarrow{QM}$ and $\overrightarrow{QO}$ ✓ reasons that $\overrightarrow{QM}$ and $\overrightarrow{QO}$ are ½ $\overrightarrow{QR}$ and ½ $\overrightarrow{QP}$ ✓ completes proof	

Alternate Solution to Q18(b)



1. Radius is perpendicular to the tangent.

$$\angle OAD = \angle ODE = 90°$$

2. Angle sum in a quadrilateral EDOA

$$\angle AOD = 180 - \angle DEA \quad (1)$$

3. Angle sum in isosceles triangle DOA

$$\angle ODA = \frac{1}{2}(180 - \angle AOD)$$
$$= 90 - \frac{1}{2}\angle AOD \quad (2)$$

4. Alternate Segment Theorem

$$\angle ODA = \angle CAB$$

Hence:

$$\begin{aligned}\angle DEA &= 180 - \angle AOD \\ &= 180 - 2(90 - \angle ODA) \\ &= 180 - 180 + 2\angle \mathrm{ODA} \\ &= 2\angle \mathrm{ODA} \\ &= 2\angle CAB \quad (3)\end{aligned}$$

Let $\angle DEA = x$

$$\angle ADO = 180 - x \quad (1)$$
$$\begin{aligned}\angle ODA &= \frac{1}{2}(180 - (180 - x)) \\ &= \frac{1}{2}(180 - 180 + x) \\ &= \frac{1}{2}x \quad (2)\end{aligned}$$
$$\angle CAB = \angle ODA = \frac{1}{2}x \quad (3)$$

Hence $\angle DEA = 2\angle ODA$

Specific behaviours

✓ diagram
✓ (1) Uses radius-tangent to determine $\angle AOD$ in relation to $\angle DEA$
✓ (2) recognises isosceles triangle and gets $\angle ODA$
✓ (3) shows $\angle DEA = 2\angle CAB$
✓ clear reasoning throughout