



Rossmoyne Senior High School

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please
place your student identification label in this box

WA student number: In figures

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In words

Circle your Teacher's Name:

Mrs Alvaro

Mr Gibbon

Ms Robinson

Mr Tan

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	91	65
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
8	6	
9	6	
10	8	
11	8	
12	8	
13	8	
14	7	
15	8	
16	6	
17	9	
18	8	
19	9	
S2 Total	91	
S2 Wt ($\times 0.7065$)	65%	

Section Two: Calculator-assumed**65% (91 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8**(6 marks)**

Four figure numbers are to be formed from the digits 1, 2, 3, 4, 5, 6.

- (a) How many different four figure numbers can be formed
- (i) if repetition of digits is allowed? (1 mark)
- (ii) without repetition of digits? (1 mark)
- (b) How many of the numbers without repetition are greater than 5423? (3 marks)
- (c) How many of the numbers without repetition are less than 5423? (1 mark)

Question 9**(6 marks)**

A classroom has a barrel containing a large number of red crayons and green crayons.

- (a) 12 crayons are randomly drawn from the barrel and placed, in the order, on a table. In terms of their colours, how many different orders are possible? (1 mark)
- (b) An empty box is to be filled with some of the crayons. What is the smallest number of crayons that should be placed in the box to guarantee that it contains at least 6 red or at least 5 green crayons? Justify your answer. (2 marks)
- (c) Each of the 11 students in the classroom will choose 4 crayons from the barrel. Use the pigeonhole principle to prove that at least 3 of the students will choose the same colour combination of crayons. (3 marks)

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Question 10

(8 marks)

Relative to boat O at anchor in a lake, four buoys A, B, C and D have the following position vectors (with distances in metres):

$$\overrightarrow{OA} = (210, -935), \quad \overrightarrow{OB} = (90, -200), \quad \overrightarrow{OC} = (330, 360), \quad \overrightarrow{OD} = (390, -515).$$

- (a) **Prove** that the quadrilateral with vertices $ABCD$ is a trapezium, but not a parallelogram. (5 marks)

- (b) Boat X motors directly from D to B with a constant velocity, taking 2 minutes and 30 seconds. Determine the velocity in component form, and hence the speed, of boat X . (3 marks)

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Question 11

(8 marks)

- (a) Which of the following statements are false?
Justify your answer by giving a suitable counterexample(s).

(3 marks)

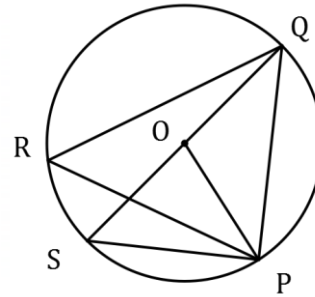
- A** $\forall x, y \in \mathbb{Z} \quad x^2 < y^2 \Rightarrow x < y$
B If $\overline{AB} = \overline{BC} \Leftrightarrow B$ is the midpoint of AC
C $\forall x \exists y: y > x$

- (b) Points P, Q, R and S lie in order on the circumference of the circle with centre O so that $PQ = 8.8$ cm, $PS = 10.5$ cm, and PR and QS are diameters. Determine, with brief reasons and to the nearest degree, the sizes of $\angle PQS$, $\angle PRS$, $\angle POS$ and $\angle RPS$. (5 marks)

Question 12

(8 marks)

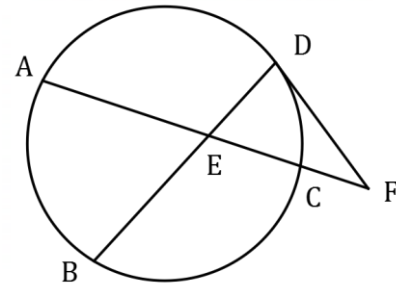
- (a) In the diagram (not to scale), points P, Q, R and S lie on a circle with centre O and diameter QS .



If $\angle QRP = 59^\circ$, determine the size of

- (i) $\angle QOP$. (1 mark)
- (ii) $\angle QSP$. (1 mark)
- (iii) $\angle SQP$. (1 mark)
- (iv) $\angle POS$. (1 mark)

- (b) In the diagram (not to scale), FD is a tangent to the circle at D . Secant AF cuts chord BD at E , and the circle at C .



$FC = 4$ cm, $CA = 21$ cm, $DF = EF$, and BE is 2 cm longer than DF .

Determine the length of DE .

(4 marks)

Question 13

(8 marks)

Two vectors are $\mathbf{a} = \begin{pmatrix} x \\ -5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 6 \\ y \end{pmatrix}$, where x and y are constants.

(a) When $x = 11$ and $y = 7$, determine

(i) $\mathbf{a} \cdot \mathbf{b}$. (1 mark)

(ii) a unit vector in the same direction as $\mathbf{a} - \mathbf{b}$, in exact form. (2 marks)

(iii) the angle between the directions of the unit vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, rounded to one decimal place. (2 marks)

(b) Determine the value of x and the value of y when $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, $x < y$, and $|\mathbf{a} + \mathbf{b}| = 12.2$. (3 marks)

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Question 14**(7 marks)**

- (a) Consider the letters in the word EXTRAVAGANCE. Determine the number of different
- (i) combinations of 4 letters chosen from the consonants in the word. (1 mark)

 - (ii) permutations of all the letters in the word. (2 marks)
- (b) Four-digit pin codes such as 0852 are made by randomly choosing four different digits from those in the number 234 567 890. Determine the fraction of all such possible pin codes that start with 2 or end in 90. (4 marks)

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Question 15**(8 marks)**

- (a) The vertices of triangle PQR lie on a circle with centre O so that PQ is a diameter. The midpoint of QR is M . Prove that OM is parallel to PR . (4 marks)

- (b) The vertices of a kite lie on the circumference of a circle. Each longer side of the kite is twice the length of the adjacent shorter side. If the area of the kite is 18 cm^2 , determine the radius of the circle. (4 marks)

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Question 16**(6 marks)**

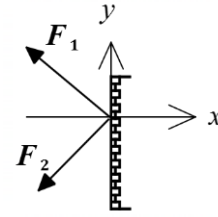
Consider the identity ${}^xC_y \times {}^yC_z = {}^xC_z \times {}^{(x-z)}C_{(y-z)}$.

- (a) With $x = 4, y = 2$ and $z = 1$, show that the left and right sides of the identity are equal. (1 mark)
- (b) State all necessary restrictions on x, y and z for the identity to exist and to be valid. (2 marks)
- (c) Prove the identity is always true, subject to all necessary restrictions. (3 marks)

Question 17

(9 marks)

The diagram at right, not to scale, shows forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting in the same vertical plane on a small hook fixed to a vertical wall. $\mathbf{F}_1$ has magnitude 147 N and acts at an angle of elevation of 22° and $\mathbf{F}_2$ has magnitude 195 N and acts at an angle of depression of 42° .



The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.

- (a) Sketch a triangle to show the relationship between $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{R}$. (1 mark)

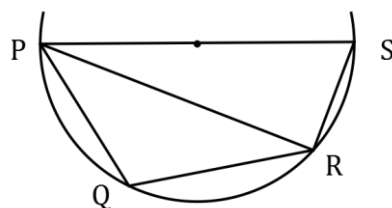
- (b) Determine, with working out, the magnitude of $\mathbf{R}$ and the acute angle it makes with the wall. (5 marks)

- (c) The wall exerts a force on the hook of equal magnitude to $\mathbf{R}$ but in the opposite direction. Express this force using unit vectors $\mathbf{i}$ and $\mathbf{j}$. (3 marks)

Question 18

(8 marks)

- (a) Points P, Q, R and S lie on an arc of a circle as shown, so that PS is a diameter.



Let $\lambda = \angle SPR$ and $\mu = \angle PQR$.

Determine in simplest form the relationship between λ and μ . Justify your answer.

(3 marks)

- (b) Tangents from E touch a circle at A and D . Diameter DC and tangent EA are both extended to meet at B . Prove that $\angle DEA = 2\angle CAB$.

(5 marks)

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Question 19**(9 marks)**

A small boat has a cruising speed of 17 km/h in still water. The boat leaves point A at 8:30 am and travels to point B , 7.7 km due east of A , where it turns and travels to point C , 3.6 km due north of B . The boat then returns to A . A current of 2.6 km/h runs in an easterly direction throughout the area, during the entire period of travel.

Determine the time at which the boat is expected to return to A .

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Supplementary page

Question number: _____

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