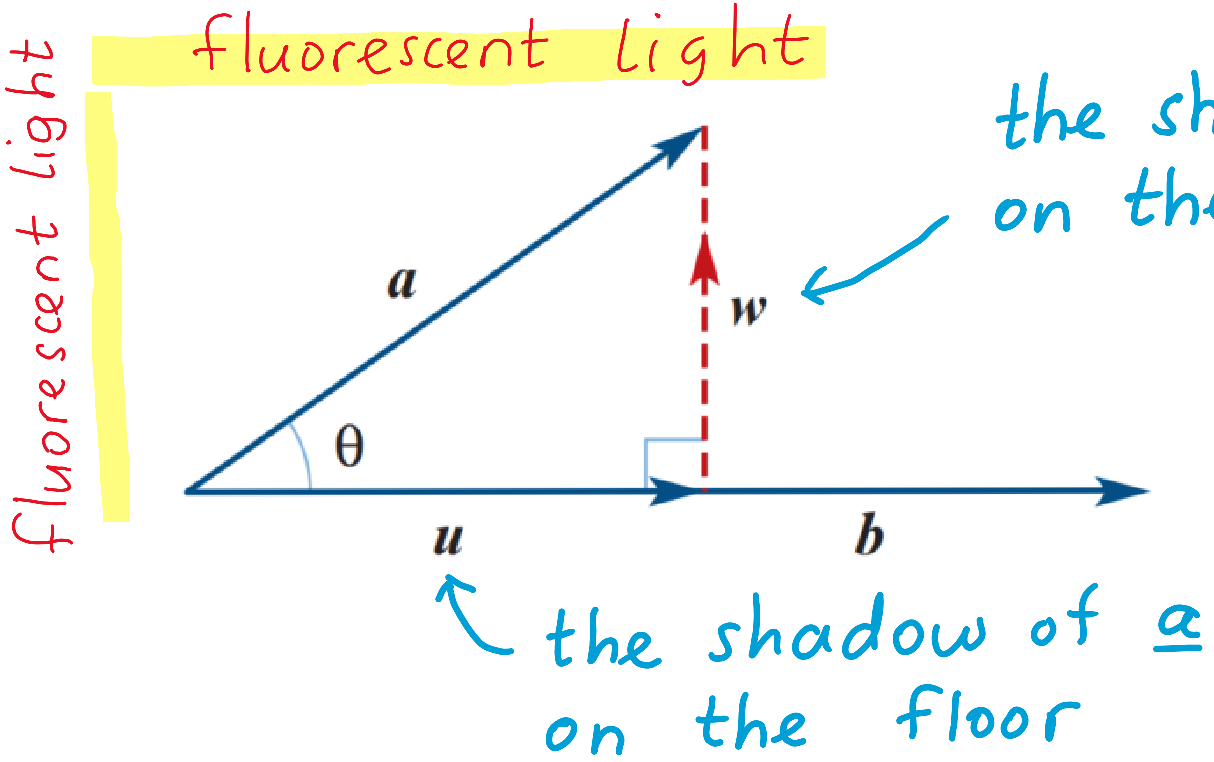
Year 11 Mathematics Specialist

Vector Projections

Ref: Cambridge Chapter 17D

Term 2 Week 2 Thursday (Double)

17D Vector projections



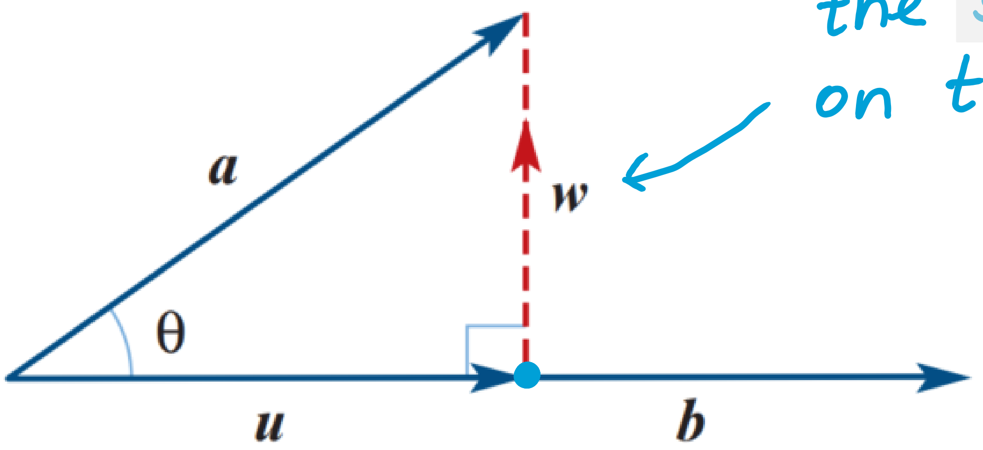
decompose $\underset{\sim}{a}$ into a sum of two vectors

1. one // to $\underset{\sim}{b}$
2. one $\perp$ to $\underset{\sim}{b}$

17D Vector projections

fluorescent light

fluorescent light



the shadow of $\underset{\sim}{a}$ on the wall

$\underset{\sim}{w} = |\underset{\sim}{a}| \sin\theta \, \underset{\sim}{j}$

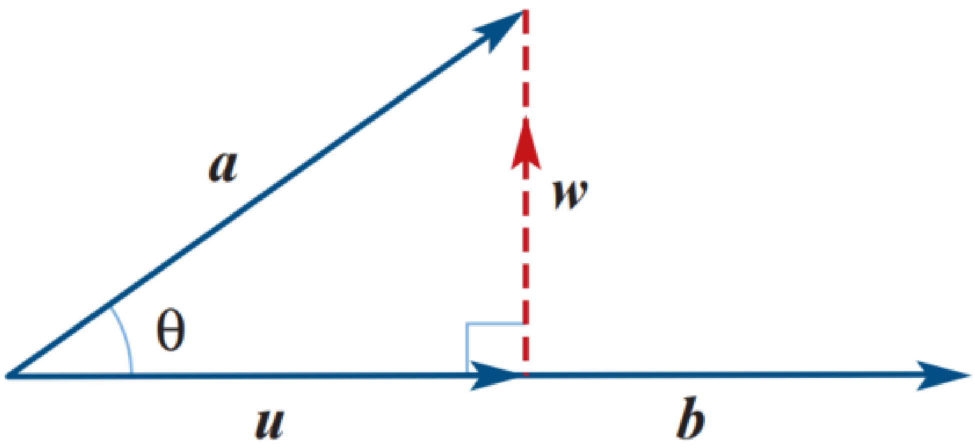
the shadow of $\underset{\sim}{a}$ on the floor

$\underset{\sim}{u} = |\underset{\sim}{a}| \cos\theta \, \underset{\sim}{i}$

u is the vector projection of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$

vector resolute

17D Vector projections



$$\underline{a} = \underline{u} + \underline{w}$$

$$\underline{u} = k\,\underline{b}$$

$$\underline{w} = \underline{a} - \underline{u}$$

$$\underline{w} = \underline{a} - k\underline{b}$$

If $\underline{w} \perp \underline{b}$

then $\underline{w} \cdot \underline{b} = 0$

$$(\underline{a} - k\underline{b}) \cdot \underline{b} = 0$$

$$\underline{a} \cdot \underline{b} - k(\underline{b} \cdot \underline{b}) = 0$$

$$\underline{a} \cdot \underline{b} = k(\underline{b} \cdot \underline{b})$$

$$k = \frac{\underline{a} \cdot \underline{b}}{\underline{b} \cdot \underline{b}}$$

$$k = \frac{\underline{a} \cdot \underline{b}}{\underline{b} \cdot \underline{b}}$$

$$\therefore \underline{u} = \frac{\underline{a} \cdot \underline{b}}{\underline{b} \cdot \underline{b}}\,\underline{b}$$

The vector resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$

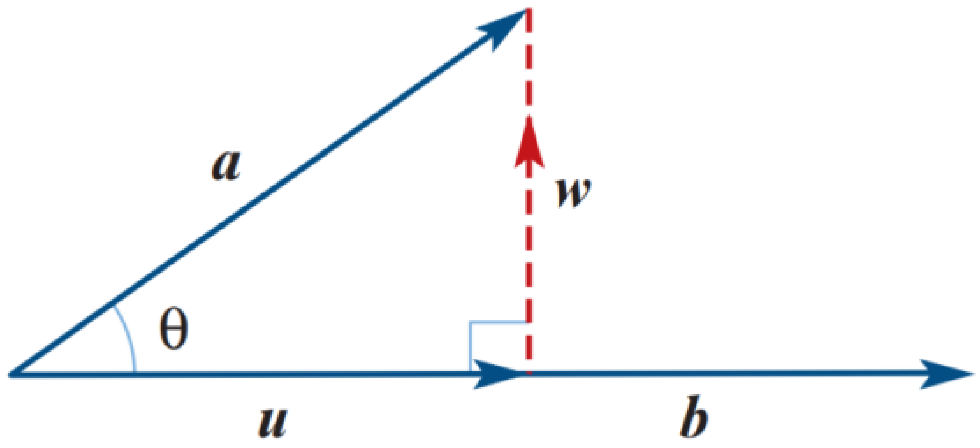
$$\underset{\sim}{u} = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\underset{\sim}{b} = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|b|^2}\underset{\sim}{b}$$

$$\underset{\sim}{a}\cdot\hat{\underset{\sim}{b}} = \underset{\sim}{a}\cdot\frac{\underset{\sim}{b}}{|b|} = \underset{\sim}{a}\cdot\underset{\sim}{b}\times\frac{1}{|\underset{\sim}{b}|}$$

$$= \frac{(\underset{\sim}{a}\cdot\underset{\sim}{b})\times\underset{\sim}{b}}{|\underset{\sim}{b}|\times|\underset{\sim}{b}|}$$

$$= \left(\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{b}|}\right)\left(\frac{\underset{\sim}{b}}{|\underset{\sim}{b}|}\right) \cdots\rightarrow \underset{\sim}{a}\cdot\underset{\sim}{b}\left(\frac{1}{|\underset{\sim}{b}|}\right)\left(\frac{\underset{\sim}{b}}{|\underset{\sim}{b}|}\right)$$

$$= (\underset{\sim}{a}\cdot\hat{\underset{\sim}{b}})\,\hat{\underset{\sim}{b}} \leftarrow\cdots \left(\underset{\sim}{a}\cdot\frac{\underset{\sim}{b}}{|\underset{\sim}{b}|}\right)\left(\frac{\underset{\sim}{b}}{|\underset{\sim}{b}|}\right)$$



$$\underset{\sim}{u} = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$$

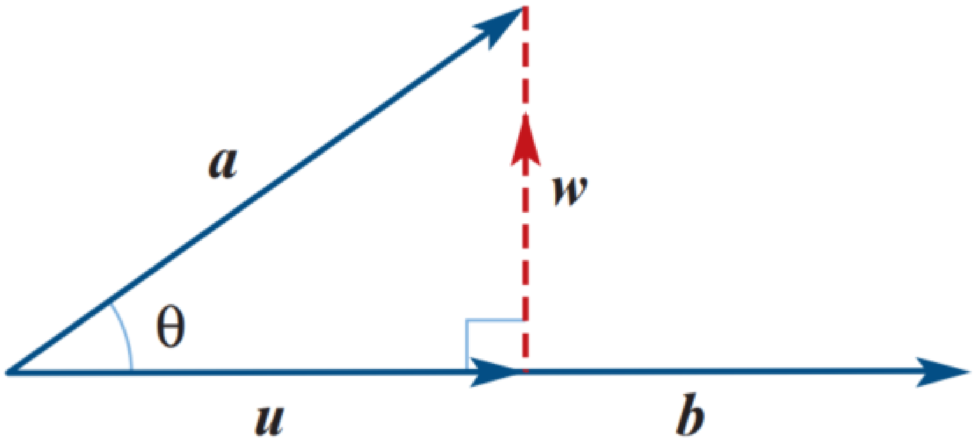
$$\underset{\sim}{u} = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|^2} \underset{\sim}{b}$$

$$\underset{\sim}{u} = (\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}}) \hat{\underset{\sim}{b}}$$

the $\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}}$ or $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|}$

is the signed length of the vector resolute $\underset{\sim}{u}$

It is also called the scalar resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$



$$\underline{u} = \overbrace{\frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}}^{\text{scalars}}\,\underline{b}$$

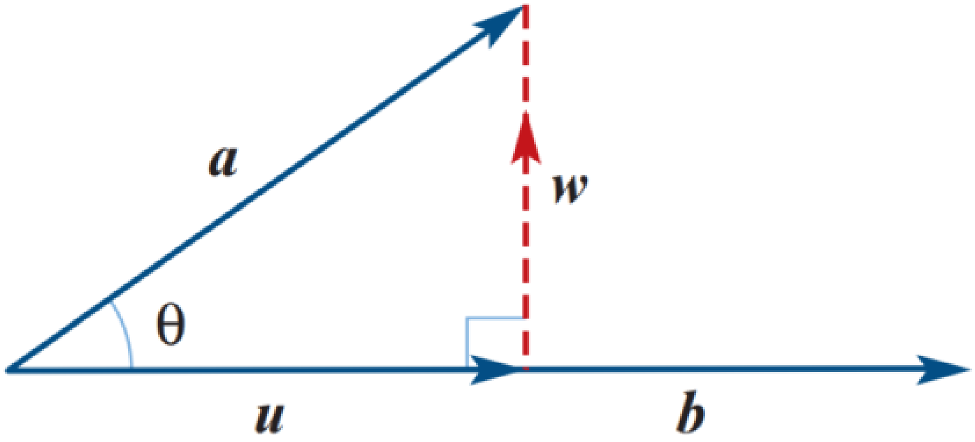
$$\underline{u} = \frac{\underline{a}\cdot\underline{b}}{|\underline{b}|^2}\,\underline{b}$$

start with $\underline{b}$ and scale it accordingly

$$\underline{u} = (\underline{a}\cdot\hat{\underline{b}})\,\underbrace{\hat{\underline{b}}}_{\text{vectors}}$$

start with $\hat{\underline{b}}$ and scale it accordingly

Remember this slide?

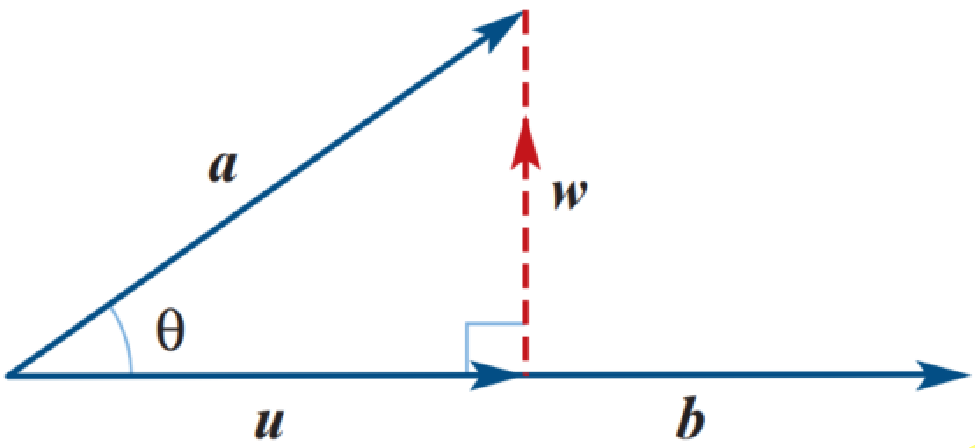


$\underline{a} = \underline{u} + \underline{w}$

$\quad \llcorner\ \underline{u} = k\,\underline{b}$

$\underline{w} = \underline{a} - \underline{u}$

Remember this slide ?



$$\underline{a} = \underline{u} + \underline{w}$$

$$\llcorner\ \underline{u} = k\,\underline{b}$$

$$\underline{w} = \underline{a} - \underline{u}$$

$$\downarrow$$

$$\underline{w} = \underline{a} - \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\,\underline{b}$$

$$\underline{a} = \underline{u} + \underline{w}$$

$$\underline{a} = \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\,\underline{b} + \left(\underline{a} - \frac{\underline{a}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\,\underline{b}\right)$$

this is called resolving vector $\underline{a}$ into rectangular components, one // to $\underline{b}$ and one $\perp$ to $\underline{b}$

Example 12

Let $\boldsymbol{a} = \boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j}$. Find the vector resolute of:

a $\boldsymbol{a}$ in the direction of $\boldsymbol{b}$

b $\boldsymbol{b}$ in the direction of $\boldsymbol{a}$.

a) Vector resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$

is $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = 1 - 3 = -2$

$\underset{\sim}{b} \cdot \underset{\sim}{b} = 1 + 1 = 2$

$= \dfrac{-2}{2}(\underset{\sim}{i} - \underset{\sim}{j})$

$= -i + j$

b) Vector resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$

is $\dfrac{\underset{\sim}{b} \cdot \underset{\sim}{a}}{\underset{\sim}{a} \cdot \underset{\sim}{a}} \underset{\sim}{a} \quad = \dfrac{-2}{10}(\underset{\sim}{i} + 3\underset{\sim}{j}) \quad = -\frac{1}{5}(\underset{\sim}{i} + 3\underset{\sim}{j})$

Example 13

Find the scalar resolute of $\boldsymbol{a} = 2\boldsymbol{i} + 2\boldsymbol{j}$ in the direction of $\boldsymbol{b} = -\boldsymbol{i} + 3\boldsymbol{j}$.

The scalar resolute of $\underline{a}$ in the direction of $\underline{b}$

is $\dfrac{\underline{a} \cdot \underline{b}}{|\underline{b}|}$

$\underline{a} \cdot \underline{b} = -2 + 6 = 4$

$|\underline{b}| = \sqrt{1+9} = \sqrt{10}$

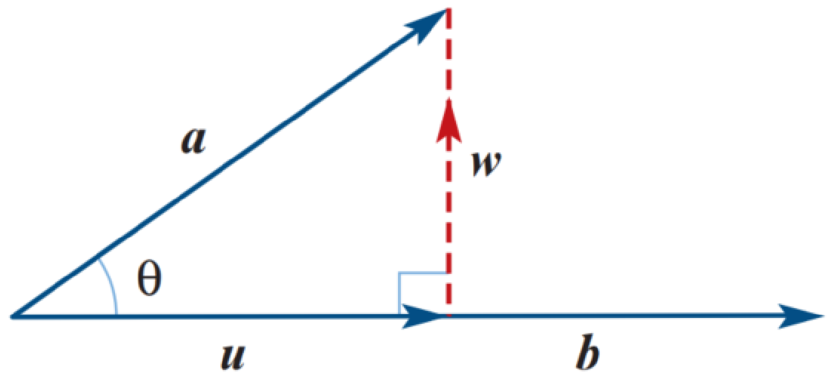
$= \dfrac{4}{\sqrt{10}}$

$= \dfrac{4}{\sqrt{10}} \dfrac{\sqrt{10}}{\sqrt{10}} = \dfrac{4\sqrt{10}}{10}$

$= \dfrac{2\sqrt{10}}{5}$

Example 14

Resolve $\boldsymbol{i} + 3\boldsymbol{j}$ into rectangular components, one of which is parallel to $2\boldsymbol{i} - 2\boldsymbol{j}$.



from this, the horz. component is $\underset{\sim}{u} = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\,\underset{\sim}{b}$

the vert. component is

$\underset{\sim}{w} = \underset{\sim}{a} - \underset{\sim}{u}$

let $\underset{\sim}{a} = \underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{b} = 2\underset{\sim}{i} - 2\underset{\sim}{j}$

$\underset{\sim}{a}\cdot\underset{\sim}{b} = 2 - 6 = -4$

$\underset{\sim}{b}\cdot\underset{\sim}{b} = 4 + 4 = 8$

vector resolute $= -\frac{4}{8}(2\underset{\sim}{i} - 2\underset{\sim}{j})$

$= -\underset{\sim}{i} + \underset{\sim}{j}$

perpendicular component is

$\underset{\sim}{a} - (-\underset{\sim}{i} + \underset{\sim}{j}) = (\underset{\sim}{i} + 3\underset{\sim}{j}) - (-\underset{\sim}{i} + \underset{\sim}{j}) = 2\underset{\sim}{i} + 2\underset{\sim}{j}$

$\therefore \underset{\sim}{i} + 3\underset{\sim}{j} = (-\underset{\sim}{i} + \underset{\sim}{j}) + (2\underset{\sim}{i} + 2\underset{\sim}{j})$